MORGAN'S MIXED HODGE STRUCTURE AND HAIN'S MIXED HODGE STRUCTURE

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We consider the following "theorem".

"Theorem". *Let M be a compact Kähler manifold and $\pi_1(M,x)$ its fundamental group. There exist mixed Hodge structures on the Malcev completion of $\pi_1(M,x)$.*

There are two ways to construct mixed Hodge structures as this "Theorem". The first way is given by Morgan ([5]) by using the Sullivan 1-minimal model. It is known that for the de Rham complex $A^*(M)$ of a manifold M there exists a unique "1-minimal" differential graded algebra $\mathcal{M}^*$ such that there is a homomorphism $\phi : \mathcal{M}^* \to A^*(M)$ inducing isomorphisms on 0-th and first cohomologies and an injection on second cohomology. Sullivan proves that $\mathcal{M}^*$ is isomorphic to the Malcev algebra $\pi_1(M,x)$. Thus Morgan gives constructions of mixed Hodge structures on $\mathcal{M}^*$.

The second way is given by Hain ([1]) by using iterated integrals. In this way, the mixed Hodge structure is "functorial" for pointed compact kähler manifolds (M,x). We can see that Morgan's mixed Hodge structure and Hain's mixed Hodge structure have the same Hodge numbers. However precise relations between them are not known.

The purpose of this talk is to give new comparisons between Morgan's mixed Hodge structure and Hain's mixed Hodge structure by using unipotent variations of mixed Hodge structure ([2]).

References

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