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Abstract

Let G be a finite group. Throughout the talk, we mean by a **manifold** a smooth manifold and by a ***G*-action on a manifold** a smooth G-action on a manifold. For a natural number m, we call a G-action on a manifold an ***m*-fixed-point action** if the G-fixed-point set consists of exactly m points. E. Stein and T. Petrie began the study of one-fixed-point actions on spheres; G. Bredon, T. Petrie, and S. Cappell–J. Shaneson began the study of two-fixed-point actions on spheres from the view point of tangential representations at fixed points. We are still interested in the next miscellaneous problem.

Problem. Let $U_1, \ldots, U_m$ be real G-modules. We wonder if there exist a sphere S with G-action such that

(1) $S^G = \{x_1, \ldots, x_m\}$, and
(2) $T_{x_i}(S) \cong_G U_i \quad (\forall\, i = 1, \ldots, m)$.

In the talk, we will report some results concerning the following problems. We call G an **Oliver group** if G can act on some disk without G-fixed points.

Problem. Let G be an Oliver group.

(1) Which dimensional spheres possess effective one-fixed-point (or odd-fixed-point) actions?

(2) Which real G-modules are isomorphic to $T_x(S)$ for some sphere S with G-action such that $S^G = \{x\}$?

(3) How large is the Smith set $Sm(G)$?