# ON COVERING HIGHER FANO MANIFOLDS BY BOTT TOWERS

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de Jong-Starr and Araujo-Castravet embarked upon a problem of generalizing the Mori's famous theorem of the uniruledness (i.e. covered by rational curves) of Fano manifolds to the $k$-ruledness (i.e. covered by rational $k$-folds) of appropriately defined higher Fano manifolds. They obtained some results for the case $k = 2$ and $k = 3$, respectively. Of these, the approach taken by Araujo-Castravet was to make use of the polarized minimal families of rational curve, preciously studied by Kollar, Miyaoka, Cho-Miyaoka-Shepherd-Barron, Kebekus, Hwang-Mok, and they appied this procedure again to deal with the case $k = 3$.

Taku Suzuki stressed the importance of how many times this construction of the polarized minimal families of rational curve can be iterated, and proved (with a little patch by Nagaoka and myself) an appropriately defined "$k$-Fano"manifolds, whose pseudo-index are reasonably large, actually admit such a $k$-iteration of polarized minimal families of rational curve.

While Araujo-Castravet, and Suzuki all state covering properties, their covering properties require some possible exceptional cases. This problem happens if we stick to cover by $\mathbb{P}^k$.

In this talk, I shall report, if we compromise to cover by more general rational $k$-folds given by (generalized) Bott tower, then we may actually cover without exception.

A motivation of this result is the classical theorem of Conte-Murre, which claims the validity of the Hodge conjecture for uniruled 4-folds. I shall discuss possible higher analogues of the Conte-Murre theorem, too.