Speaker : Naoki Kitazawa (Institute of Mathematics for Industry, Kyushu University)
Title : Representing various differentiable manifolds via explicit fold maps
Abstract : Morse functions exist densely on smooth manifolds. From singular points, appearing discretely, we can know homology groups and several information on homotopy of the manifolds. For example, the existence of a Morse function with just 2 singular points, a generalization of the natural height function of an unit sphere, characterizes a sphere topologically except 4-dimensional cases. The theory had been established in the former half of the 20th century and contributed to the development of algebraic and differential topology of higher dimensional manifolds in 1950–70s. For example, Milnor applied the theory to show that an exotic 7-dimensional sphere is topologically a sphere.

Later, as branches of a higher dimensional version of the theory, studies of singularity and geometric theory of fold maps and general generic smooth maps have been developed. These studies were started by Thom and Whitney in 1950s: a fundamental study of generic smooth maps on manifolds whose dimensions are larger than 2 into the plane. This was later studied by Levine and Eliashberg has launched and solved problems on the existence of fold maps into general Euclidean spaces. Later, especially since 1990s, Saeki and Sakuma have studied algebraic and differential topological properties of fold maps satisfying good conditions and manifolds admitting such maps. For example, for fundamental manifolds such as homotopy spheres and manifolds obtained by iterations of several fundamental procedures starting from homotopy spheres such as sphere bundles over spheres, the differences of topologies and differentiable structures have been shown to be closely related with the differences of classes of fold maps the manifolds admit. As a simplest example, exotic spheres often do not admit special generic maps, which are defined as higher dimensional versions of Morse functions with just 2 singular points (standard spheres always admit such maps).

The speaker has demonstrated related studies. For example, the author has explicitly constructed explicit fold maps on various explicit manifolds. Different from knowing the existence of fold maps on manifolds, constructing explicit maps on explicit manifolds are in general difficult even on fundamental manifolds as before. Through these works, one of interesting results is the success of obtaining and representing higher dimensional manifolds by explicit fold maps. They were difficult to represent in geometric or combinatorial ways.

This talk is mainly on these constructive studies. If we can, then we would like to discuss relations with topics on the transformation group theory, for example, on symmetries of these manifolds. Manifolds obtained by constructing the maps seem to be symmetric in various senses. However, they also seem to be difficult to handle in combinatorial manners as several classes of manifolds (as the author, who has not so much knowledge about symmetries, thinks).